And this being so, then, if q now denotes any one of the $6n-3$ coordinates, each of the remaining equations assumes the form

$$\frac{d}{dt}\cdot\frac{dT'}{d\dot{q}}-\frac{dT'}{dq}=-\frac{dV}{dq},$$

viz., we have thus $6n-3$ equations for the relative motion of the bodies of the system.

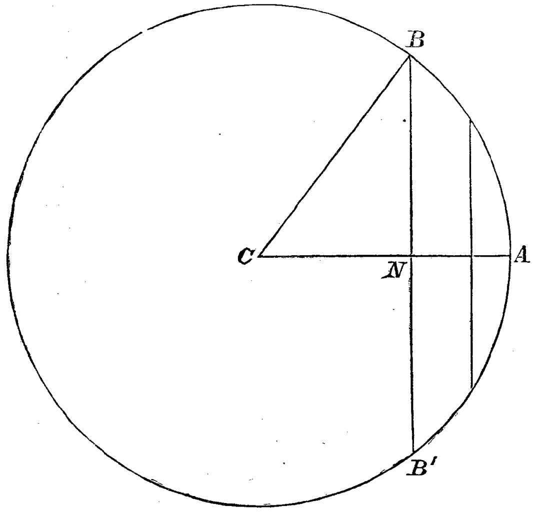
On Photographing Solar Transits by the use of the "Starlit Transit Eye-piece" formerly described; and other methods. By Dr. Royston-Pigott, M.A., F.R.S.

It may perhaps be recollected that the eye-piece contains a ruled micrometer, displaying equidistant parallel lines drawn on a silver film with great accuracy. At present the Sun transits in a five-feet telescope from line to line in five seconds—a second per foot of focal length.

If the image of the Sun in transits be projected on a screen of paper, a peculiar phenomenon is observed worth noting. So soon as the solar light illuminates the field, the transparent lines remain *black* until the sun flashes across the lines.

On the instant that the limb makes contact, a bright line is seen rapidly lengthening as a brilliant tangent, becoming a chord to the disk.

During the passage of the Sun across the bars, at each minute change of position the phases of the bars pass through a continual series of varying patterns.



If now an instantaneous photograph be taken at a known instant of time, the measured length of any one of these brilliant

chords, and—still better—the measured lengths of several, may be employed accurately to determine the instant the Sun's centre would transit each particular bar.

Let C be the Sun's centre approaching the brilliant bar BB′ at the precise mean time M. Let BB′ be measured on the screen placed at a known fixed distance from the object-glass, so that the apparent diameter of the Sun may be assignable on the screen; so as to compare the length of the bar BB′ with the diameter of the Sun.

Then the time which will elapse before the centre of the Sun will transit over BB′ is (r being the semi-diameter of the Sun in time, θ the angle BCA, and $\frac{1}{2}\Delta$ being half the length of the bar)

$$t = \mathrm{CN} = r \cos \theta.$$

Also

$$\sin \theta = \frac{\mathrm{BN}}{\mathrm{BC}} = \frac{\frac{1}{2}\Delta}{r}.$$

Then the time of transit of Sun's centre over BB′ will be $\mathrm{M}+t$.

If several bars be measured on the photograph, say three

$$\mathrm{T}, \text{ the mean time, } = \frac{1}{3}(t_1 + t_2 + t_3),$$

or

$$\mathrm{T} = \frac{r}{3} \cdot (\cos \theta_1 + \cos \theta_2 + \cos \theta_3);$$

and the final result for the time of passage of Sun's centre is $\mathrm{M}+\mathrm{T}$.

Should some of the bars be to the West and the others to the East, $\cos \theta$ will have contrary signs algebraically.

Or if there are n bars measured, then, according to the usual notation,

$$\mathrm{T} = \frac{r}{n} \Sigma (\cos \theta).$$

Now, admitting that the lines are ruled very accurately (a thing not approached by mechanical arrangement of spider lines), any one of these lines should give the same instant of transit, because the centre passes each after equal intervals. By inspecting the tables of natural sines and tangents, it appears that, up to the first 18 minutes of arc, the tangents and sines are identical up to the $\frac{1}{10{,}000{,}000}$th of the radius, and the minutes of arc up to the semi-diameter of the Sun are equal within the

$\frac{1}{10,000,000}$th of the radius. From this it follows that the measurements of the bars, if drawn equidistant, for a telescope of 100 inches focus will not exhibit a tangential error exceeding the $\frac{1}{10,000}$th of an inch. At the full breadth of the image of the Sun—32 minutes—the error would be no greater. But as the field of view would probably not much exceed the semi-diameter, the tangential error from employing equidistant lines may be neglected.

The difficulty of ascertaining on the screen the precise magnitude of the Sun's semi-diameter may be avoided by using a screen at a standard distance from the focal point of the object-glass. And if a single uncemented triplet be used to throw the image upon the screen, whose focal length for simplicity is 1 inch, say at a distance of 10 inches from it, the usual formulæ for optical foci will give

$$\frac{1}{10}+\frac{1}{v}=\frac{1}{1}, \quad \text{or} \quad v=\frac{10}{9},$$

and the magnifying power on the screen will be

$$\frac{u}{v}=10\div\frac{10}{9}=9,$$

so that, at the distance of 10 inches from the eye-lens of the telescope (with a 1-inch triplet), the disk of the Sun will be magnified exactly *nine* times.

The actual size of the disk of the Sun at the focus of the telescope is readily obtained from the equation

$$D = F \sin \delta,$$

F being its focal length and δ the diameter of the Sun.

If the focal length be 60 inches, for example, and the Sun's diameter be 32′ 20″·2, the diameter on the screen will be

$$\delta = 9F \sin 32' \; 20''{\cdot}2$$

$$\log \delta = \begin{cases} \log 9 & = 0{\cdot}95424 \\ +\log 60 & = 1{\cdot}77815 \\ +\log 32' \; 20''{\cdot}2 & = 7{\cdot}97334 \end{cases}$$

$$\text{(rejecting the index)} \qquad 10{\cdot}70573 = 5{\cdot}078 \text{ inches}$$

By a slight shifting of the screen, the Sun's image for the day may be made exactly to correspond with 5 inches. Should that be too large a photograph, then a telescope of shorter length can be used. At 6 inches distance the image will be magnified five

times, just one time less than the number of inches of the screen from the lens, as seen from the formula

$$\frac{1}{d}+\frac{1}{v}=1,$$

for then

$$v=\frac{d}{d-1},$$

and the diameter of the Sun will be 2·821 inches. At 3 inches it will be magnified twice, and show a disk of diameter 1·128 inches. This very useful law for a 1-inch lens gives at once the magnifying power at any distance; say 30 inches distance from the lens, the image will be 29 times greater than the object in its focus.

Doubtless many methods will suggest themselves for utilising the appearance of the brilliant solar bars during observations for a transit. Suppose the photograph taken shortly before the time of central transit, so that the whole system of equidistant parallel bars are illuminated: the most favourable bar will evidently give the most exact position of the Sun's centre and the time of transit, viz. the bar or bars which give the smallest chords; for here they increase most rapidly.

Should the observer calculate the times from several bars, and take the mean of each, I presume this mean would scarcely differ from that obtained from the most favourable bar.

In photographing, the effect of obliquity of vision or parallax by displacement of the eye from the true optical axis is avoided. Also another error, irradiation, will so nearly affect both the disk of the Sun and the extremities of the brilliant bars as to lessen this source of error, which for *Venus* is about, I believe, half a second, and probably not much more for the Sun. Now half a second will not be much on the screen, and at each end of the bar this will make a whole second; and on a screen placed at 9 inches, this will with the telescope in question be one four-hundredth of an inch, and for a disk 2 inches in diameter it will be reduced to a thousandth. I do not therefore apprehend irradiation will be a serious error, if neglected. But it will be measurable by comparing a maximum bar with the Sun's calculated diameter on the screen.

In the case where a second photograph is taken after central transit, there will probably be great choice of verification bars. This method does not necessarily require the Sun-picture to be taken at the instant of transit.

But if such a system were preferred, a thin plate of glass placed at right angles to the axis, between the lens and the screen, and close behind the former, would both transmit the picture and reflect it to the observer in parallel rays by a suitable lens. The observer could then, by a spring, flash the photograph at the most favourable instant.

Supposing several bars are then chosen, avoiding those which

are formed nearly equal to the diameter of the Sun, the determination of the distance of the Sun's centre from each of the chosen bars will be much facilitated by using the natural cosines of θ after determining the value of θ from the equation

$$\sin\theta = \frac{\frac{1}{2}\Delta}{r} = \frac{\text{half a bar}}{\text{rad. of Sun}}.$$

The foregoing equation

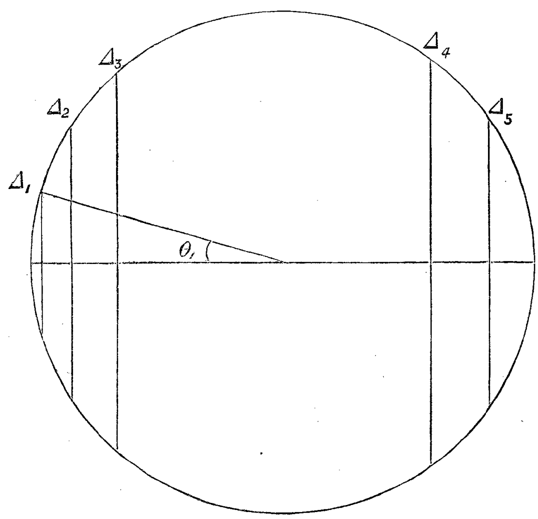
$$\mathrm{T} = \frac{r}{n}\,\Sigma\,(\cos\theta)$$

will then give the time of transit of the Sun's centre, and the sign of the result, whether positive or negative, will show whether the time of observation is before or after the true transit. Moreover, should the first numerical differences not be exactly constant between $\cos\theta_1$, $\cos\theta_2$, $\cos\theta_3$, &c., two things may happen: either the lines are not ruled precisely equidistant, or else the measurement of the bars is untrue. For, since $t = r\cos\theta$, and r is constant, t varies as $\cos\theta$, so that $\cos\theta$ must increase exactly *pari passu* with the time the Sun has passed, or will reach a given bar.

After taking the sum of the natural cosines arithmetically, the final value of $\pm\mathrm{T} = \frac{r}{n} \times$ sum of the cosines will of course be readily found by logarithms.

Probably sufficient accuracy (as perhaps diminishing the effect of irradiation) will be attained by preferring bars forming chords nearly subtending 60°, or equal to the Sun's radius.

A trigonometrical example may possibly interest some of our Fellows, and I therefore beg to submit one.



Let the bars

Δ_1	Δ_2	Δ_3	Δ_4	Δ_5
+18·2	+25·6	40·2	40	32·2

be measured; determine the value of the times of transit of the Sun's centre over each bar: $(r = 32{\cdot}68)$

$$\sin\theta_1 = \Delta_1 \div 2r = \frac{18{\cdot}2}{2 \times 32{\cdot}68} = \frac{9{\cdot}1}{32{\cdot}68}$$

$$\log\sin\theta_1 = \left\{\begin{array}{r} {\cdot}9590 \\ -1{\cdot}5143 \end{array}\right\} = 9{\cdot}4447, \quad \theta_1 = 16^\circ\ 10'$$

$$\log\cos\theta_1 = 9{\cdot}9825$$

$$\log r\cos\theta_1 = \left\{\begin{array}{r} 1{\cdot}5143 \\ +9{\cdot}9825 \end{array}\right\} = 1{\cdot}4968, \quad t_1 = \underline{31^{s}{\cdot}39}$$

$$\sin\theta_2 = \frac{12{\cdot}8}{32{\cdot}68}, \quad \log\sin\theta_2 = \left\{\begin{array}{r} 1{\cdot}10721 \\ -1{\cdot}51428 \end{array}\right\} = 9{\cdot}59293, \quad \theta_2 = \sin 23^\circ\ 4'$$

$$\left.\begin{array}{rr} \log\cos\theta_2 = & 9{\cdot}9638 \\ +\log r = & +1{\cdot}5143 \end{array}\right\} = 11{\cdot}4781, \quad t_2 = \underline{30^{s}{\cdot}07}$$

$$\sin\theta_3 = \frac{20{\cdot}1}{32{\cdot}68}, \quad \log\sin\theta_3 = \left\{\begin{array}{r} 1{\cdot}3031 \\ -1{\cdot}5143 \end{array}\right\} = 9{\cdot}7888, \quad \theta_3 = 37^\circ\ 57'$$

$$t_3 = r\cos\theta_3 \left\{\begin{array}{rr} \log\cos\theta_3 = & 9{\cdot}8968 \\ +\log r = & +1{\cdot}5143 \end{array}\right\} = 1{\cdot}4111, \quad t_3 = \underline{25^{s}{\cdot}77}$$

$$\sin\theta_4 = \frac{20}{32{\cdot}68}, \quad \log\sin\theta_4 = \left\{\begin{array}{r} 1{\cdot}3010 \\ -1{\cdot}5143 \end{array}\right\} = 9{\cdot}7867, \quad \theta_4 = 37^\circ\ 44'$$

$$t_4 = r\cos\theta_4 \left\{\begin{array}{rr} \log\cos\theta_4 = & 9{\cdot}8981 \\ +\log r = & +1{\cdot}5143 \end{array}\right\} = 11{\cdot}4124, \quad t_4 = \underline{25^{s}{\cdot}85}$$

$$\sin\theta_5 = \frac{16{\cdot}1}{32{\cdot}68}, \quad \log\sin\theta_5 = \left\{\begin{array}{r} 1{\cdot}2068 \\ -1{\cdot}5143 \end{array}\right\} = 9{\cdot}6925, \quad \theta_5 = 29^\circ\ 31'$$

$$\left.\begin{array}{rr} \log\cos\theta_5 = & 9{\cdot}9396 \\ \log r = & +1{\cdot}5143 \end{array}\right\} = 1{\cdot}4539, \quad t_5 = \underline{28^{s}{\cdot}44}$$

The bars Δ_1, Δ_2, Δ_3 lie east of Sun's centre and are negative. Times of Sun's centre passing the

Bars	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	{if M be M.T. of transit.
M ±	$-31^{s}{\cdot}39$	$-30^{s}{\cdot}07$	$-25^{s}{\cdot}77$	$25{\cdot}85$	$28{\cdot}45$	
Differences	$1^{s}{\cdot}32$	$4^{s}{\cdot}30$	$51^{s}{\cdot}62$	$2^{s}{\cdot}60$		

By the microscope the intervals of each can be accurately measured with ease to the 10,000th of an inch, as also the width of the cuts. By a low power, the photograph would probably yield the constant of irradiation for the given telescope, which, however, will probably vary with greater or less brilliancy of the Sun, and states of the atmosphere.

The extremity of each bar will be projected outwards by the irradiation; while the breadth of a bar will be determined by mixed optical causes, such as diffraction and interference, as the peculiar darkness of the bright bands before illumination by the Sun particularly denotes. Again, the known diameter of the Sun compared with its image will be another method for determining readily this error.

I offer this method with great diffidence to the Society, having myself no practical knowledge of photography. Nor am I able to offer any valuable opinion whether this method of silver-film bars for transits is of any advantage. For myself, I have been contented with watching the very beautiful appearances of the transit of the Sun across these film bars, either projected on a screen or directly with a darkened eye-piece.

The method of direct observation by projection appears to have some advantages. The screen can readily be sketched with an exact copy of the illuminated bars, and divided into a scale of parts; the observer, at a given tick of the clock, can mark with a pencil the precise position of the extremity of a given bar. The diameter of the Sun may easily be enlarged to a foot. The lens spoken of as a triplet is an extraordinarily fine one, by Wray, formed of three glasses cemented to each other with balsam, and capable of displaying a very fine image of the Sun at 10 inches.

Should the solar heat melt the balsam, Mr. Wray can very readily construct these triplets without balsam. And if one of the interior surfaces is faintly blued with silver, the glass becomes a useful and beautifully defining eye-piece, though of small field. A large shade of pasteboard slipped over the object end of the telescope greatly improves the splendour and distinctness of the projected image of the Sun; and a number of transits across the bars, by the commencing flash and length, can be taken with great ease and comfort by the observer, the eye being entirely saved the heat, glare, and strain usual to the generally adopted method of spider lines. But, I again beg to repeat, I feel quite incompetent to form any opinion of the real value or utility of the methods now described.

I would add (while correcting the proof) that if the silver film be just the sufficient thickness or opacity to permit the outline of the limb of the Sun to be photographed, it is probable that nearly the whole of the irradiation of the limb may thus be destroyed, leaving that of the extremities of the bars in full irradiatory display.

The time can also afterwards, by the same translucent bars be compared with a star transit.